AN OVERVIEW OF THERMODYNAMIC GAS CYCLES WORKING AT QUANTUM DEGENERACY CONDITIONS

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ABSTRACT

An overview of various thermodynamic gas cycles working at quantum degeneracy conditions is presented. It is well known that an ideal gas deviates from its classical behaviour under quantum degeneracy conditions (sufficiently low-temperature or high-density conditions). Although the gas is still an ideal gas, it obeys Bose-Einstein or Fermi-Dirac statistics instead of Maxwell-Boltzmann statistics. Under quantum degeneracy conditions, the corrected equation of state is valid instead of the classical ideal gas equation of state. The corrected equation of state is obtained by considering the quantum degeneracy of gas particles and it is reduced to classical ideal gas equation of state at the classical ideal gas conditions (sufficiently high-temperature or low-density conditions). In thermodynamic analyses of ideal gas cycles, efficiency (except the Carnot efficiency) and work expressions are derived by using the classical ideal gas equation of state and some generalisations are obtained by using the results of these analyses. Analyses of gas cycles working with ideal Bose and Fermi gases allow us to investigate how these expressions and generalisations are effected by the quantum degeneracy. In literature, various gas cycles working with ideal Bose (^4He) and Fermi (^3He) gases have been thermodynamically analysed at quantum degeneracy conditions. Here, heat, work and entropy expressions for isothermal, isobaric, isochoric and isentropic processes under the quantum degeneracy conditions are introduced. The behaviour of Carnot and Ericsson power and Brayton refrigeration cycles working with ideal Fermi and Bose gas are reviewed by using these expressions. For these cycles, advantage or disadvantage of the use of Fermi and Bose gases are also given in a brief summary. This overview provides a general picture of the physics of the cycles working under quantum degeneracy conditions.

KEYWORDS : Ideal quantum gas cycles, Quantum degeneracy, Work, Efficiency, Coefficient of performance.

1. INTRODUCTION

Recently, analyses of quantum heat engines have become one of the interesting research subjects for the people working on thermodynamics and statistical physics. Many works have been made on the finite-time thermodynamic analyses of various quantum heat engines in literature [1-14]. Also, equilibrium thermodynamic analyses of some heat engines working with ideal Bose and Fermi gases have been worked [15-21].

In thermodynamic analyses of ideal gas cycles, efficiency (except the Carnot efficiency) and work expressions are derived by using the classical ideal gas equation of state or some other equations based on it. From the results of these analyses, some generalisations about the behaviours of gas cycles are obtained [22, 23]. Analyses of gas cycles working with ideal Bose and Fermi gases allow us to generalise the thermodynamic models of gas cycles and to investigate how behaviours of these cycles are effected by the quantum degeneracy of working fluid.

It is well known that an ideal gas deviates from classical ideal gas behaviour under sufficiently low-temperature or high-density gas conditions. This deviation results purely from the quantum mechanical degeneracy of gas particles. Therefore, these gas conditions are called here Quantum Degeneracy Conditions (QDC). Although the gas is still an ideal gas, it obeys Bose-Einstein or Fermi-Dirac statistics instead of Maxwell-Boltzmann statistics. In literature, these types of ideal gases are called ideal quantum gases [21]. Under QDC, the corrected equation of state is valid instead of the classical ideal gas equation of state. The corrected equation of state contains the quantum degeneracy of gas particles and it is reduced to classical equation of state at high-temperature or low-density gas conditions (Classical Gas Conditions, CGC). Thus, the corrected equation of state is a more general equation of state which is valid under both QDC and CGC.

In this work, an overview of thermodynamic gas cycles working at quantum degeneracy conditions is presented. The analyses of Carnot and Ericsson power cycles and Brayton refrigeration cycle working with ideal Bose (^{4}He) and Fermi (^{3}He) gases are reviewed briefly. By using the corrected equation of state, derivation of heat, work and entropy exchange expressions for constant temperature, constant volume and constant pressure processes are introduced. Some remarkable results relating to thermodynamic analysis of the considered cycles are given.

2. BASIC CONCEPTS

2.1 Derivation of the Corrected Ideal Gas Equation of State for a Monatomic Ideal Gas [15-21]

Expressions for number density and pressure of monatomic ideal Bose and Fermi gases are obtained as follows after some mathematical manipulations on their general descriptions [24-27]

$$n = \pm\lambda (k_b T)^{3/2} Li_{3/2}\left[\pm \exp(\mu / k_b T)\right] \qquad (1)$$

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$$P = \pm\lambda (k_b T)^{5/2} Li_{5/2}\left[\pm \exp(\mu / k_b T)\right] \quad (2)$$

where k_b is the Boltzmann's constant, T is gas temperature, μ is chemical potential, $Li_l(x)$ is the Polylogarithm function defined as $Li_z(x) = \sum_{l=1}^{\infty} x^l / l^z$ [28], and λ is defined as,

$$\lambda = \frac{(2\pi m_o)^{3/2} g}{h^3} \quad (3)$$

where g is the number of possible spin orientations of a gas particle, h is the Planck's constant and m_o is the rest mass of gas particles. In equations (1) and (2), the upper signs are used for Bose gas and the lower signs for Fermi gas. Corrected equation of state for an ideal gas can be written as follows by using equations (1) and (2)

$$P = nk_b T\, CF(\mu / k_b T) \quad (4)$$

where $CF(\mu / k_b T)$ is called as the correction factor and it is defined for a Fermi gas as follows

$$CF_F(\mu / k_b T) = \frac{Li_{5/2}\left[-\exp(\mu / k_b T)\right]}{Li_{3/2}\left[-\exp(\mu / k_b T)\right]}. \quad (5)$$

For a Bose gas, the number density of particles in equation (4) should be equal to the number density of particles having energy greater than the zero translational energy [24-27]. Hence, the Bose-Einstein condensation should be considered when the correction factor is defined for a Bose gas. Consequently, the correction factor for a Bose gas can be expressed as

$$T > T_o \Rightarrow CF_B(\mu / k_b T) = \frac{Li_{5/2}\left[\exp(\mu / k_b T)\right]}{Li_{3/2}\left[\exp(\mu / k_b T)\right]} \quad (6a)$$

$$T \le T_o \Rightarrow CF_B(\mu / k_b T) = 0.5135 \left(\frac{T}{T_o}\right)^{3/2} \quad (6b)$$

where T_o is the Bose-Einstein condensation temperature, which is defined in terms of pressure, P, as

$$T_o(P) = \frac{P^{2/5}}{k_b}\left(\frac{1}{\lambda 2.612 \times 0.5135}\right)^{2/5} \quad (7a)$$

and in terms of number density as

$$T_o(n) = \frac{n^{2/3}}{k_b}\left(\frac{1}{\lambda 2.612}\right)^{2/3}. \quad (7b)$$

The quantity of $\mu / k_b T$ is calculated from equation (1) implicitly, as a function of (n, T) or from equation (2) as a function of (P, T). Therefore, correction factor can be determined for given (n, T) or (P, T) parameters.

In the classical gas region, in which the gas behaves like a Maxwellian gas, the correction factors are reduced to unity since $\mu << -k_b T$. On the other hand, in the case of completely degenerate gas state, the correction factor for Fermi gas can be simplified as

$$CF_F \approx \frac{2}{5}\frac{T_F}{T} + \frac{\pi^2}{4}\frac{T}{T_F} \quad (8)$$

where T_F is the Fermi temperature [24-27] and it can be rearranged in terms of P as follows

$$T_F(P) = \left(\frac{15\pi^{1/2}}{8\lambda}\right)^{2/5} \frac{P^{2/5}}{k_b} \quad (9a)$$

and in terms of n as

$$T_F(n) = \left(\frac{3\pi^{1/2} g^{1/2}}{4\lambda}\right)^{2/3} \frac{n^{2/3}}{k_b}. \quad (9b)$$

For the same case, the correction factor for Bose gas can also be simplified as in equation (6b). The condition for completely degenerate gas state is ensured when $T << T_F(P)$ for Fermi gas and $T \le T_o(P)$ for Bose gas. On the other hand, the conditions for classical gas behaviour can be expressed as $T >> T_F(P)$ for Fermi gas and $T >> T_o(P)$ for Bose gas.

For a constant pressure value, variations of the correction factors with the temperature are shown in Figure1. When the temperature is high enough, both of the correction factors go to the unity. If the temperature is low enough, the correction factor for Bose gas has the values lower than the unity and it reaches to the lowest value (0.5135) when the temperature is equal to the Bose-Einstein condensation temperature, T_o. T_o is the lowest temperature for a given pressure. Although the chemical potential of a Bose gas is restricted by zero, there is no restriction for the chemical potential of a Fermi gas. Therefore, the correction factor for Fermi gas goes to infinity when the temperature goes to zero.

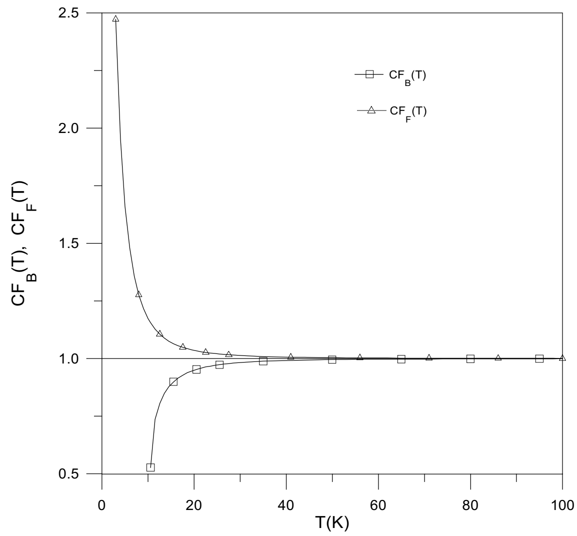

Figure 1. Variation of the quantities of CF_B and CF_F with temperature for a given pressure of 10^7 Pa.

In Figure 2, dependence of the correction factors on the pressure is given for a constant temperature. For a Bose gas, there is an upper limit of the pressure and it is determined from equation (7a). For the pressure values higher than P_o, the constant temperature condition is violated [25]. For Fermi gas, there is no such a limitation.

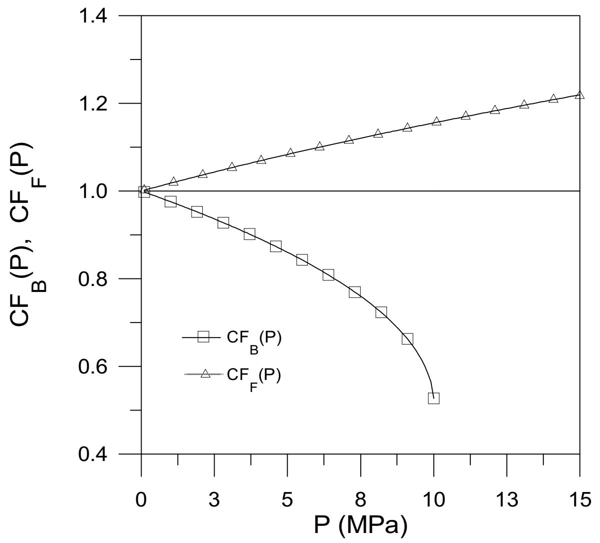

Figure 2. Variation of the quantities of CF_B and CF_F with pressure for a given temperature of 10.53 K.

2.2 Derivation of Heat, Work and Entropy Exchange Expressions for Common Reversible Processes of Closed Systems [19]

By using the corrected equation of state, equation (4), heat, work and entropy exchange expressions for a monatomic ideal gas can be derived for different conditions as follows,

i) Constant Temperature Condition

$$Q_{if}^{T} = mRT\left[\frac{5}{2}\left[CF(P_f,T)-CF(P_i,T)\right]-\int_i^f \frac{CF(P,T)}{P}dP\right] \quad (10a)$$

$$Q_{if}^{T} = mRT\left[\frac{3}{2}\left[CF(V_f,T)-CF(V_i,T)\right]+\int_i^f \frac{CF(V,T)}{V}dV\right] \quad (10b)$$

$$W_{if}^{T} = mRT\left[CF(P_f,T)-CF(P_i,T)-\int_i^f \frac{CF(P,T)}{P}dP\right] \quad (11a)$$

$$W_{if}^{T} = mRT\int_i^f \frac{CF(V,T)}{V}dV \quad (11b)$$

$$S_{if}^{T} = \frac{Q_{if}^{T}}{T} \quad (12)$$

ii) Constant Pressure Condition

$$Q_{if}^{P} = \frac{5}{2}mR\left[T_f CF(P,T_f)-T_i CF(P,T_i)\right] \quad (13)$$

$$W_{if}^{P} = \frac{2}{5}Q_{if}^{P} \quad (14)$$

$$S_{if}^{P} = \frac{5}{2}mR\left[CF(P,T_f)-CF(P,T_i)+\int_i^f \frac{CF(P,T)}{T}dT\right] \quad (15)$$

iii) Constant Volume Condition

$$Q_{if}^{V} = \frac{3}{2}mR\left[T_f CF(V,T_f)-T_i CF(V,T_i)\right] \quad (16)$$

$$S_{if}^{V} = \frac{3}{2}mR\left[CF(V,T_f)-CF(V,T_i)+\int_i^f \frac{CF(V,T)}{T}dT\right] \quad (17)$$

iv) Constant Entropy Condition

Since the ratio μ/T remains constant during an isentropic process [25], quantum and classical ideal gases have the same isentropic relations. Therefore, work expression can be obtained as

$$W_{if}^{S} = \frac{P_f V_f - P_i V_i}{1-k} = \frac{mR\, CF_{const.}}{1-k}\left(T_f - T_i\right) \quad (18)$$

where $k = 5/3$ for monatomic ideal gas and

$$CF_{const.} = CF(V_i,T_i) = CF(V_f,T_f) = CF(P_i,T_i) = CF(P_f,T_f) \quad (19)$$

3. SOME APPLICATIONS ON GAS CYCLES WORKING AT QUANTUM DEGENERACY CONDITIONS

3.1 Work Analysis of Carnot Power Cycle [16]

Carnot power cycles working with Bose, Fermi and classical ideal gases are called Bose, Fermi and classical Carnot cycles, respectively. Since Carnot efficiency (η_C) is independent from gas properties, net work outputs of Bose and Fermi Carnot cycles (W_B and W_F) can be expressed as $W_B = \eta_C Q_H^B$ and $W_F = \eta_C Q_H^F$. Therefore, they can be derived by using equation (10b). By dividing these works to the work of classical Carnot cycle (W_C), work ratios are defined as $R_W^B = W_B/W_C$ and $R_W^F = W_F/W_C$. The following expression is obtained for R_W^B and R_W^F,

$$R_W^j = \frac{1}{\ln\left(r_v \tau^{1/k-1}\right)}\left[\int_{v_L}^{v_H \tau^{1/k-1}} \frac{CF_j(T_H,v)}{v}dv + \frac{3}{2}\left[CF_j\left(T_H, v_H \tau^{1/k-1}\right)-CF_j\left(T_H,v_L\right)\right]\right]. \quad (20)$$

Variations of R_W^B and R_W^F with T_H are shown in Figure 3. It is seen that R_W^B and R_W^F go to unity (as can be expected) for high values of T_H since the quantum degeneracy becomes unimportant. In this case, working gas always stays a classical ideal gas throughout the cycle and correction factors are approximately equal to unity. Thus, net work output is approximately equal to that of a classical Carnot cycle for high values of T_H. On the other hand, it can be seen that R_W^B takes the values greater than unity for some lower values of T_H, whereas

R_W^F always takes the values lower than unity for all values of T_H.

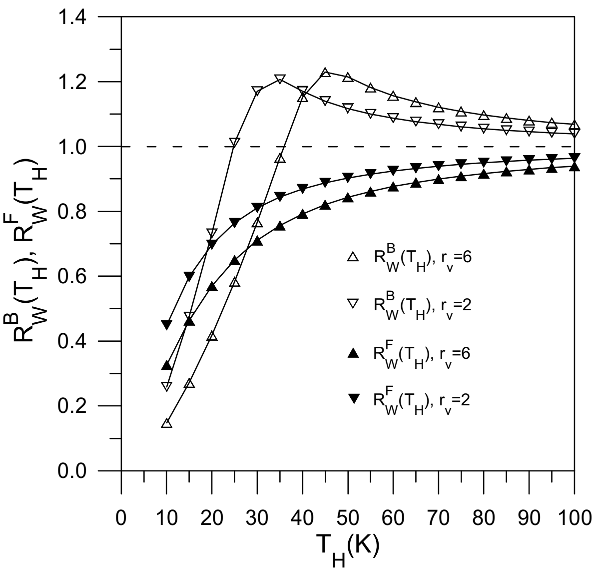

Figure 3. Variations of R_W^B and R_W^F versus to T_H. Figure parameters v_H^B, v_H^F and τ are chosen as $v_H^B = 0.00069$ m^3/kg, $v_H^F = 0.00067$ m^3/kg and $\tau = 0.4$.

If $T_0(v_L) \geq T_0(v_H) \geq T_H > T_L$ then, the working gas is always completely degenerate quantum gas throughout the cycle. In this case, R_W^B can be simplified by using equations (6b) and (7b) in equation (20) and by considering the identity of $n = 1/v\, m_o$ as follows,

$$R_W^B \cong 0.5135 \frac{r_v \tau^{1/k-1} - 1}{\ln(\tau^{1/k-1} r_v)} \left[\frac{T_H}{T_0(v_L)}\right]^{3/2} \frac{5}{2}. \qquad (21)$$

An upper limit for maximum value of R_W^B can be determined as follows

$$\left(R_W^B\right)_{Max} = \underset{\substack{\tau \to 1 \\ r_v \to 1 \\ T_H \to T_0(v_L)}}{Limit} \left(R_W^B\right) \cong 1.2825. \qquad (22)$$

As shown in Figure 3, R_W^B has a maximum value greater than unity and lower than the upper limit given by equation (22).

Under QDC, the use of Bose gas is advantageous for a Carnot power cycle when T_H is above a critical temperature value. It is not possible to give an analytical expression for this critical temperature. However, it can be numerically calculated by equalising equation (20) to the unity and solving T_H value from this equation for given τ, r_v and v_H values. In Figure 3, this critical temperature is 25 K for r_v=2 and 35 K for r_v=6. At the temperatures below this critical temperature value, the use of Bose gas becomes disadvantageous. On the other hand, the use of Fermi gas is always disadvantageous for a Carnot power cycle. For Carnot refrigeration and heat pump cycles, the use of Bose gas still is advantageous since it increases the pumped heat per cycle for the temperatures higher than the critical temperature. However, the use of Fermi gas is still disadvantageous for these cycles since it decreases the pumped heat per cycle.

3.2 Efficiency Analysis of Ericsson Power Cycle [15]

The Ericsson cycle involves two isothermal and two isobaric heat exchange processes. Isothermal processes consist of heat addition (at T_H) and heat rejection (at T_L) processes between the working gas and heat reservoirs. Whereas isobaric heat addition (at P_H) and rejection (at P_L) processes take place in the regenerator and there is no need extra heat addition from or rejection to a heat reservoir [22-23]. When the correction factors are different from the unity, however, this is not true and the extra heat addition to or rejection from the regenerator becomes necessary. The amounts of heats in isothermal processes are also changed in this case. Hence, the energy balance and so the efficiencies of Bose Ericsson and Fermi Ericsson cycles are different from that of the classical Ericsson cycle.

For a Bose Ericsson cycle, the lowest temperature of the cycle (T_L) is restricted by the highest pressure of the cycle (P_H). This restriction results from the Bose-Einstein condensation phenomena under the constant pressure condition. Temperature of the gas (T) should not be lower than the condensation temperature (T_o), since this case causes violation of the constant pressure condition. Therefore the lowest value of T_L is equal to $T_o(P_H)$ and it can be calculated by means of equation (7a) as $T_L^{Min} = T_0(P_H)$. Similarly, it is possible to calculate the highest value of P_H for a given value of T_L. This pressure is the Bose-Einstein condensation pressure (P_o) for a given temperature and it is easily obtained from the equation (7a) as $P_H^{Max} = P_o(T_L)$.

The amounts of total heat input and output of the cycle depend on the energy balance in the regenerator. The processes in the regenerator are isobaric processes. For Fermi and Bose monatomic ideal gases, the specific heat at constant pressure can be written as follows

$$c_P^j(T,P) = \frac{5}{2} R \frac{d}{dT}\left[T \text{ x } CF_j(T,P)\right]. \qquad (23)$$

The variation of $c_P^j(T,P)/R$ with the temperature can be seen in Figure 4 for different pressure value. From the figure, one can written the followings,

$$c_P^B(T,P_H) > c_P^B(T,P_L) \qquad (24)$$

$$c_P^F(T,P_H) < c_P^F(T,P_L). \qquad (25)$$

By means of the equations (24) and (25), it can be easily seen that $\left|Q_{P_H}^B\right| > \left|Q_{P_L}^B\right|$ and $\left|Q_{P_H}^F\right| < \left|Q_{P_L}^F\right|$. When the temperature is high enough, these inequalities become the equalities since the correction factors go to unity. This result can be seen from the equation (13). In this case, heat addition to or rejection from the regenerator is not necessary and the classical Ericsson cycle is obtained.

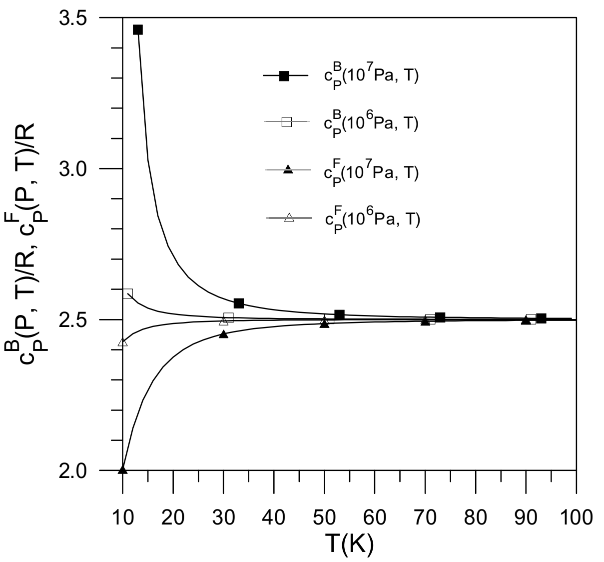

Figure 4. Variation of the quantities of $c_P^B(P,T)/R$ and $c_P^F(P,T)/R$ with temperature for two different pressures, (10^6 and 10^7 Pa).

In a Bose cycle, heat addition to the regenerator is necessary since $\left|Q_{P_H}^B\right| > \left|Q_{P_L}^B\right|$. Thus, the efficiency of Bose Ericsson cycle can be expressed by using equations (10a) and (13) as

$$\eta_B^{Ericsson} = 1 - \frac{Q_{T_L}^B}{Q_{P_H}^B - Q_{P_L}^B + Q_{T_H}^B} = 1 - \tau \times \frac{\frac{5}{2}\left[CF_B(T_L,P_L) - CF_B(T_L,P_H)\right] + \int_{P_L}^{P_H} \frac{CF_B(T_L,P)}{P}dP}{\frac{5}{2}\tau\left[CF_B(T_L,P_L) - CF_B(T_L,P_H)\right] + \int_{P_L}^{P_H} \frac{CF_B(T_H,P)}{P}dP} \quad (26)$$

On the other hand, heat rejection from the regenerator is necessary in a Fermi Ericsson cycle since $\left|Q_{P_H}^F\right| < \left|Q_{P_L}^F\right|$. Thus the efficiency of Fermi Ericsson cycle is obtained by using equations (10a) and (13) as follows

$$\eta_F^{Ericsson} = 1 - \frac{Q_{T_L}^F + Q_{P_L}^F - Q_{P_H}^F}{Q_{T_H}^F} = 1 - \tau \times \frac{\frac{5}{2}\frac{1}{\tau}\left[CF_F(T_H,P_L) - CF_F(T_H,P_H)\right] + \int_{P_L}^{P_H} \frac{CF_F(T_L,P)}{P}dP}{\frac{5}{2}\left[CF_F(T_H,P_L) - CF_F(T_H,P_H)\right] + \int_{P_L}^{P_H} \frac{CF_F(T_H,P)}{P}dP} \quad (27)$$

If the working gas remains the classical gas throughout the cycle, it can be easily seen that all the correction factors go to unity and the efficiencies defined by equations (26) and (27) go to Carnot efficiency (as expected).

Variations of the efficiencies of Bose, Fermi and the classical Ericsson cycles with the temperature ratio, τ, are comparatively shown in Figure 5 for two different pressure ratios, $r_P = P_H / P_L$.

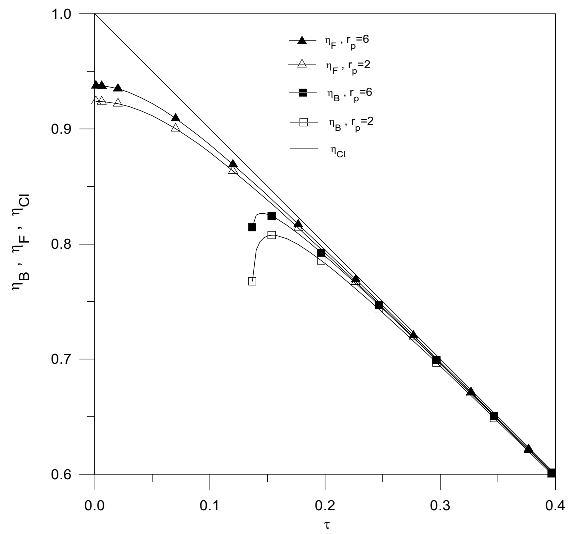

Figure 5. Variation of the efficiencies of Bose, Fermi and the classical Ericsson cycles with the temperatures ratio, τ, for two different pressure ratios. The highest temperature and pressure of the cycle are chosen as $T_H = 77$ K and P_H=10^7 Pa respectively.

In this figure, the value of T_H is chosen as 77 K and τ depends on T_L only. Similarly, the value of P_H is chosen as 10^7 Pa and different values of r_p correspond to different P_L values. At the conditions of $T_H > T_L >> T_o(P_H)$ for Bose cycle and $T_H > T_L >> T_F(P_H)$ for Fermi cycle, working gas is always in the classical gas state throughout the cycle. Therefore the efficiencies of Bose, Fermi and the classical cycles are approximately the same, $\eta_F \approx \eta_B \approx \eta_{Cl}$, since the correction factors are very close to unity. This situation is seen in Figure 3 for high values of τ. At the conditions of $T_H >> T_L = T_o(P_H)$ for Bose cycle and $\{T_H >> T_F(P_H),\ T_F(P_L) >> T_L\}$ for Fermi cycle, working gas is a classical gas near T_H while it is a completely degenerate gas near T_L. In this case, the efficiency of Fermi cycle goes to its maximum value and it can be expressed in the following simple form by using equations (8), (9a) and (27)

$$\left(\eta_F^{Ericsson}\right)^{Max} = 1 - \tau \frac{2}{5}\left(\frac{15}{4\lambda}\right)^{2/5} \frac{1}{k_b T_L} \frac{P_H^{2/5} - P_L^{2/5}}{\ln\left(P_H^{2/5}\right) - \ln\left(P_L^{2/5}\right)} \quad (28)$$

$$\left(\eta_F^{Ericsson}\right)^{Max} = 1 - \frac{T_F\left(P_H\right)}{T_H}\frac{1 - r_P^{-2/5}}{\ln\left(r_P\right)} . \qquad (29)$$

Although the efficiency of the classical cycle goes to unity while τ goes to zero, the efficiency of Fermi cycle goes to its upper limit, which is lower than the unity. Equation (29) describes this upper limit and it is seen in Figure 5. For Bose cycle, the minimum value of τ is restricted by equation (7a) as $\tau_{Min} = T_0(P_H)/T_H$. On the contrary of Fermi cycle, in this limit case, the efficiency expression of Bose cycle can not be expressed in a more simple form than the equation (26). Although η_{Cl} continuously increases with the decreasing τ value, it is seen that there is a maximum for η_B . Also, order of the efficiencies is that $\eta_{Cl} > \eta_F > \eta_B$ and it is conserved for all values of τ.

Consequently, it can be said that it is not always a good method to decrease the lowest temperature (T_L) of the cycle in order to improve the efficiency. This method is useful if the state of gas is sufficiently far from the degeneracy state. Moreover, Bose-Einstein condensation restricts the lowest temperature of a Bose cycle for a given value of P_H. Whereas there is no such a limitation in a Fermi cycle. Additionally, there is no an optimum value for the lowest temperature in Fermi cycle. However, the efficiency of a Fermi cycle goes to a finite value lower than the unity when τ goes to zero.

3.3 Refrigeration Load Analysis of Brayton Refrigeration Cycle [21]

It can be easily shown that coefficient of performance (*COP*) value of Brayton refrigeration cycle is not effected by the quantum degeneracy of refrigerant. The reason of this result is that the correction factor remains constant during the isentropic change of state.

The refrigeration load ratio can be defined as follows

$$R_{Q_L}^j = \frac{Q_L^j}{Q_L^C} \qquad (30)$$

where Q_L^C is refrigeration load of the classical Brayton cycle. By using equation (13) and isentropic gas relations and considering that the correction factors are equal to unity for classical cycle, equation (30) can be written as

$$R_{Q_L}^j = \frac{CF_j\left(P_H,T_H\right) - CF_j\left(P_L,T_L\right)\tau r_P^{2/5}}{1 - \tau r_P^{2/5}} . \qquad (31)$$

In Figure 6, it is shown that how $R_{Q_L}^B$ and $R_{Q_L}^F$ depend on T_L. For high values of T_L, $R_{Q_L}^B$ and $R_{Q_L}^F$ go to unity. This is an expected behaviour since the refrigerant becomes classical ideal gas and correction factors go to unity for high values of T_L. It is seen that $R_{Q_L}^B$ is always greater than unity while $R_{Q_L}^F$ is lower than unity. Therefore, refrigeration load of Bose Brayton cycle is greater than or equal to that of the Classical Brayton cycle while there is an opposite situation for Fermi Brayton cycle, $Q_L^B \geq Q_L^C \geq Q_L^F$. At the temperatures lower than T_o, volume dependence of the pressure of a Bose gas vanishes and it depends on only temperature. This situation can easily be seen by using equations (6b) and (7b) in equation (4). At the temperatures below T_o, therefore, constant pressure condition means the constant temperature condition. For this reason, in a Bose Brayton cycle, the minimum value of T_L is restricted by $T_o\left(P_L\right)$, which can be calculated by equation (7b). This restriction is also seen in Figure 4. On the other hand, there is no such a restriction in case of Fermi gas. Since P_H is taken as a constant, the value of P_L and thus the quantum degeneracy decrease with increasing value of r_P. Therefore, the curves for $r_P = 3$ are closer to unity in comparison with the curves for $r_P = 2$.

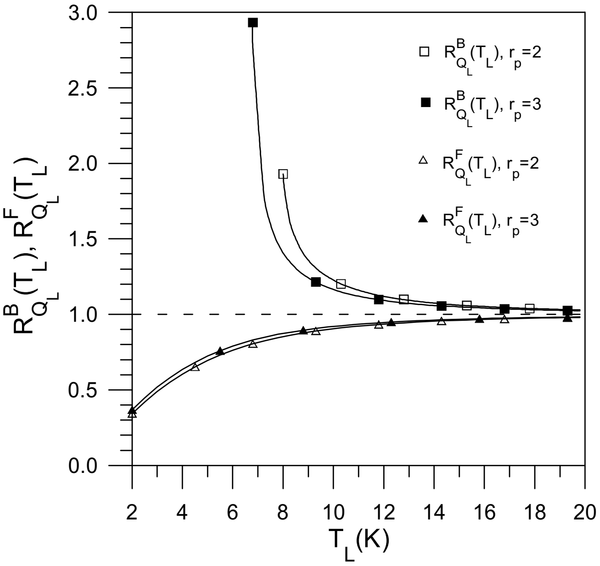

Figure 6. Variations of $R_{Q_L}^B$ and $R_{Q_L}^F$ versus to T_L. The values of P_H and τ are chosen as $P_H = 10^7$ Pa and τ=0.6.

An increment/decrement in the value of refrigeration load per cycle corresponds to an increment/decrement in the value of work input per cycle since COP value remains constant for a given r_p value. Thus, in a Brayton refrigeration cycle using a Bose type refrigerant, more heat is absorbed per cycle from the cold reservoir by consuming more work per cycle. On the other hand, in a Brayton refrigeration cycle using a Fermi type refrigerant, less heat is absorbed per cycle from the cold reservoir by consuming less work per cycle. Lifetime of a refrigerator mainly depends on the mechanical wear. Mechanical wear increases with increasing number of cycle. In a real refrigerator, furthermore, lost work also increases and COP decreases with increasing number of cycle due to the losses depending on the friction. Consequently, it can be said that Bose type of ideal gas is always advantageous for Brayton refrigeration cycle since the same refrigeration load is obtained with less number of cycle. On the contrary, Fermi type of ideal gas is always disadvantageous because it causes an increment in the number of cycle for a given refrigeration load. These results may be used in the construction of a Brayton refrigeration cycle working at quantum degeneracy conditions.

CONCLUSION

In summary, by using the results of present analysis in preceding sections it is understood that the quantum degeneracy of working gas can lead a decrement in the efficiency of gas cycles. However, efficiency is not effected by the quantum degeneracy for the cycles with four branches, if two of the branches are isentropic. On the other hand, quantum degeneracy of Bose gas causes to increment in the net work output or refrigeration load per cycle while quantum degeneracy of Fermi gas causes to a decrement in these quantities.

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NOMENCLATURE

c_P^B : Specific heat constant pressure of Bose gas.
c_P^F : Specific heat constant pressure of Fermi gas.
CF : Correction factor for classical ideal gas equation of state.
CF_B : Correction factor for ideal Bose gas.
CF_F : Correction factor for ideal Fermi gas.
COP : Coefficient of performance.
g : Number of possible spin orientations of the gas particle.
h : Planck's constant (Js).
k : Specific heat ratio.
k_b : Boltzmann's constant (J/K).
Li : The Polylogarithm function.
m : Gas mass (kg).
m_o : Rest mass of the gas particle (kg).
n : Number density of gas particles (m^{-3}).
y : Indices representing P, T or V.
P : Pressure (Pa).
P_L : The lowest pressure (Pa).
P_H : The highest pressure (Pa).
Q_L^j : Refrigeration load of the Brayton cycle working j type gas.
Q_L^C : Refrigeration load of the classical Brayton cycle.
$Q_{P_H}^j$: Heat exchange for j type gas at constant P_H.
$Q_{P_L}^j$: Heat exchange for j type gas at constant P_L.
$Q_{T_H}^j$: Heat exchange for j type gas at constant T_H.
$Q_{T_L}^j$: Heat exchange for j type gas at constant T_L.
Q_{if}^y : Heat exchange under constant y property condition.
r_p : Pressure ratio defined as $r_v = P_H / P_L$.
r_v : Specific volume ratio of the cycle defined as $r_v = v_H / v_L$.
R : Gas constant (J kg^{-1} K^{-1}).
R_w^B : Ratio of work of Bose Carnot cycle to that of classical Carnot cycle.
R_w^F : Ratio of work of Fermi Carnot cycle to that of classical Carnot cycle.
S : Entropy (J/K)
S_{if}^y : Entropy exchange under constant y property condition.
T_F : Fermi temperature (K).
T_H : The highest temperature (K).
T_L : The lowest temperature (K).
T_o : Bose-Einstein condensation temperature (K).
v : Specific volume (m^{-3} kg).
v_H : The highest specific volume (m^{-3} kg).
v_L : The lowest specific volume (m^{-3} kg).
W_B : Cycle work of Bose Carnot cycle (J).
W_C : Cycle work of classical Carnot cycle (J).
W_F : Cycle work of Fermi Carnot cycle (J).
W_{if}^y : Work exchange under constant y property condition.
η_c : Carnot efficiency.
$\eta_B^{Ericsson}$: Efficiency of Ericsson power cycle working with ideal Bose gas.
$\eta_F^{Ericsson}$: Efficiency of Ericsson power cycle working with ideal Fermi gas.
μ : Chemical potential of a gas particle (J).
τ : Temperature ratio defined as $\tau = T_L / T_H$.

Sub/superscript

f : Refers to final state.
i : Refers to initial state.
j : Refers to Bose or Fermi gas.