

THERMODYNAMIC ANALYSES OF HEAT ENGINES BASED ON THE CASIMIR EFFECT AT FINITE TEMPERATURE

A. Şişman¹ and H. Saygın^{1,2}

¹ Istanbul Technical University, Nuclear Energy Institute, 80626-Maslak, Istanbul, Turkey,

² Istanbul Technical University, Informatics Institute, 80626-Maslak, Istanbul, Turkey.

ABSTRACT

There is an attractive force between two parallel neutral conducting plates at a distance d , even at absolute zero temperature and no matter is present in the gap separating them. This effect is called the Casimir effect and it is due to the quantum vacuum fluctuations of the zero point electromagnetic field. At finite temperatures ($T \neq 0$ K), pressure and also the other thermodynamic properties become temperature dependent besides the distance d . Hence, it is theoretically possible to design some heat engines based on the Casimir effect at finite temperature. In this work, thermodynamic analysis of a Carnot heat engine based on the Casimir effect at finite temperature is aimed to understand the thermodynamic structure of this kind of machines. Two parallels, neutrals, perfectly conducting plates are considered as a heat engine device. For low ($T_C = \pi\hbar c/k_b d \gg T_H > T_L$) and high ($T_H > T_L \gg T_C$) temperature limits, heat and work exchange expressions for isothermal, isobaric, isochoric and isentropic processes are derived by taking the Casimir effect into account. It is seen that working fluid is the classical (three-dimensional) black-body radiation (BBR) at high temperature limit, while it is two-dimensional BBR at low temperature limit. By using the derived expressions, maximum work and maximum work density of a Carnot power cycle are calculated. Numerical results are compared at both high and low temperature limits. It is shown that maximum work at low temperature limit, at which the Casimir effect becomes dominant, is much less than the maximum work at high temperature limit. On the other hand, opposite of this situation is valid for maximum work density. These results provides to understand the behaviours of heat engines based on the Casimir effect and they can also be used if such heat engines based on the nano-technology are designed in the near future.

KEYWORDS

Casimir effect, Heat engines, Quantum vacuum, Zero point energy, Nano-technology.

1. INTRODUCTION

There is an attractive force between two parallel neutral conducting plates even if temperature equals to absolute zero and the gap separating them is empty. This effect, which is called the Casimir effect, arises from the quantum vacuum fluctuations of zero point electromagnetic field [1-3]. The empty space between the plates is filled zero point electromagnetic field in all wavelengths and the value of maximum wavelength is limited by two times of the distance between the plates. Therefore, an energy difference occurs between the inner and outer regions of the plates. This energy difference causes a negative (attractive) pressure, which is given by

$$p = -\frac{\pi^2 \hbar c}{240 d^4} \quad (1)$$

where d is distance between the plates, $\hbar$ is the reduced Planck's constant and c is the speed of light in vacuum [1,2,4,5]. The energy of the system constituted by plates is also given as

$$U = -\frac{\pi^2 \hbar c}{720 d^3} A. \quad (2)$$

The Casimir effect is verified by the various experiments [1,4]. Although this pressure is about 10^{-4} atm at a separation of 100 nm, it is about 1 atm at 10 nm. Thus, this effect becomes important especially in micron and submicron structures and so in mesoscopic science and nano-technology. In a rectangular or spherical cavity surrounded by perfectly conductive surface, Casimir force becomes repulsive [6-9]. Therefore, the direction of this force depends on the geometry of the cavity.

At finite temperatures ($T \neq 0$ K), the pressure and also the other thermodynamic properties become temperature-dependent besides the distance d . Therefore, theoretically it is possible to design some heat engines based on the Casimir effect at finite temperature. Recently, thermodynamics of the vacuum and engine cycles based on the Casimir effect have become an interesting research subjects for scientists who are interest in both thermodynamics and quantum theory of vacuum [3-5,10].

In this work, thermodynamic analysis of a Carnot heat engine based on the Casimir effect at finite temperature is made to understand the thermodynamic structure of this kind of machines. Two parallel neutral perfectly conducting plates are considered as a heat engine device.

2. HEAT ENGINES BASED ON THE CASIMIR EFFECT AT FINITE TEMPERATURE

A simple heat engine based on the Casimir effect at finite temperature is shown in Figure 1. One of the plates is fixed and the other is axially mobile. The value of surface area of the mobile plate, A , takes the values between the high, A_H , and low, A_L , area values. Surface area of the fixed plate is A_H . Therefore, both surface area and distance between the plates can be changed.

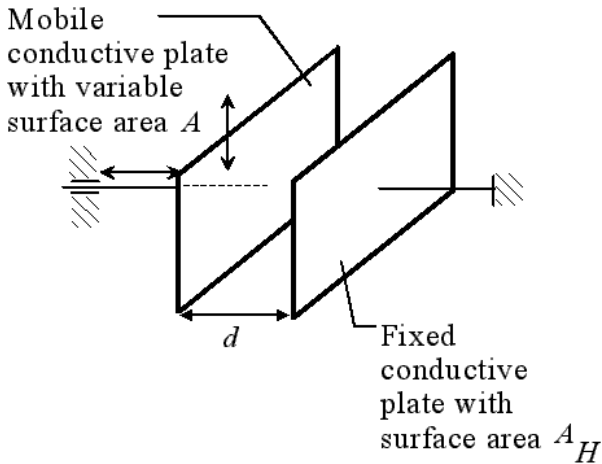


Figure 1: A simple heat engine based on the Casimir effect at finite temperature.

Because of the conservative nature of the electromagnetic field, net work output of a heat engine based on the Casimir effect at zero temperature ($T=0$) equals to zero [3]. Work is consumed on the system against the lateral forces to decrease the surface area, A and against the normal forces to increase the distance d . This causes the energy of the system given by equation (2) to increase. This process corresponds to compression process. On the contrary, when the surface area, A increases and the distance, d decreases, the work is produced by the system. This causes energy of the system to decrease. This process corresponds to expansion process. Consequently, since the consumed and produced works equal to each other, net work output equals to zero in a cyclic process for $T=0$.

To obtain a positive net work output, it is needed to another parameter effecting the Casimir pressure. Dielectric constant of the plates and temperature of the inner region between the plates are this kind of parameters. If the conversion of heat energy to mechanical energy is required, the suitable parameter is temperature. As mentioned above, Casimir effect depends on the temperature of the region between plates. Therefore, positive net work output can be obtained by operating different parts of the cycle at different temperatures. This type of a machine is a heat engine based on the Casimir effect at finite temperature. It is possible to design many different heat engines based on the Casimir effect in different geometry.

To understand the thermodynamic structure of heat engines based on the Casimir effect, a Carnot heat engine of this type is examined in this work.

2.1 Physical principle of the operation of a Carnot heat engine based on the Casimir effect at finite temperature

If there is one conductive plate with temperature, T , in empty space, it radiates in all wavelengths of thermal spectrum. If a second parallel conductive plate is brought to the distance, d , from the first plate, electromagnetic emissions in wavelengths longer than $2d$ are forbidden in the inner region between the plates. Therefore, a pressure difference occurs between the inner and outer regions. This means that net pressure is negative and the plates being attracted to each other. The total energy of electromagnetic waves in forbidden wavelengths is equal to the energy corresponding to the work done by negative pressure when the second plate is brought from infinity to distance d . For this reason, to increase the distance d , a work is done on the system. The increment of d causes a decrement in the number of forbidden wavelengths and the consumed energy on the system is used to re-create the electromagnetic waves in forbidden wavelengths. Similarly, to decrease the surface area of the plates, an amount of energy is consumed. This consumed energy is used to re-create the electromagnetic waves forbidden wavelengths in the region becoming unconstrained.

Consequently, there are two ways to make a work exchange with the system. One is to change the distance d and the second is to change the surface area A .

There are two asymptotic operation limits, namely low and high temperature limits. In the low temperature limit, the forbidden wavelengths constitute the significant part of the spectral energy distribution. Electromagnetic emissions, which are normal direction to the plates, are forbidden almost completely. This means that two-dimensional BBR emission takes place in this case. In the high temperature limit, the forbidden wavelengths constitute the insignificant part of the spectral energy distribution. Practically, the emission of BBR takes place in three-dimension in this case.

3. DERIVATION OF HEAT AND WORK EXCHANGE EXPRESSIONS DUE TO THE CASIMIR FORCE AT FINITE TEMPERATURE

Some thermodynamic properties of a system consisting of two perfectly conductive parallel plates effected by the Casimir force have been derived in literature [11]. If a critical temperature is defined as

$$T_c = \frac{\pi \hbar c}{k_b d}, \quad (3)$$

thermodynamic quantities can be expressed by approximate analytical functions for low ($T \ll T_c$) and high ($T \gg T_c$) temperature limits after some mathematical manipulations [11]. To ignore the lost

work due to external pressure, it is assumed that external pressure is equal to zero. Therefore, outer region of the plates is a vacuum with zero temperature ($T=0$) although the inner region between the plates is a vacuum at finite-temperature ($T \neq 0$). Since the temperature of outer region is assumed to be zero, pressure, energy and entropy expressions are obtained for $T < T_C$ as follows respectively,

$$p \cong -\frac{\pi^2 \hbar c}{240d^4}, \quad (4)$$

$$U \cong A \left(-\frac{\pi^2 \hbar c}{720d^3} + \frac{\xi(3)k_b^3 T^3}{\pi \hbar^2 c^2} \right), \quad (5)$$

$$S \cong \frac{3}{2} \frac{\xi(3)k_b^3}{\pi \hbar^2 c^2} AT^2 \quad (6)$$

where $\xi(x)$ is the Riemann zeta-function defined as $\sum_{n=1}^{\infty} 1/n^x$ and $\xi(3) \cong 1.202$. Equation (4) represents the zero temperature Casimir pressure. Thus, it is understood that the pressure of BBR in the inner region is negligible in comparison with the zero temperature Casimir pressure when $T < T_C$. The first term in equation (5) results from the work done by zero temperature Casimir force while the plates come from the infinity to distance d . In other word, it represents the zero temperature Casimir potential energy and it can be obtained by integrating the equation (4) from d to infinity. It can be interpreted that the second term in equation (5) represents the energy of 2D-BBR in the inner region. To verify this interpretation, the following way can be used.

For 3D-BBR, the energy is calculated by [12]

$$U = V \frac{2}{(2\pi\hbar)^3} \int_0^{\infty} \frac{cpd^3 \mathbf{p}}{\exp(cp/k_b T) - 1} \quad (7)$$

where V is volume of the considered space, p is the momentum of a photon and $d^3 \mathbf{p}$ is the volume of differential momentum space. The factor of 2 results from the fact that there are two possible transverse polarization states of an electromagnetic wave in 3D. Using the identity of $d^3 \mathbf{p} \rightarrow 4\pi p^2 dp$ and introducing $x = cp/k_b T$, equation (7) becomes,

$$U = V \frac{8\pi k_b^4 T^4}{(2\pi\hbar)^3} \int_0^{\infty} \frac{x^3 dx}{\exp(x) - 1} = \frac{\pi^2 k_b^4}{15\hbar^3 c^3} VT^4 \quad (8)$$

Equation (8) is a well-known expression for the energy of BBR in 3D [12]. To calculate the energy of 2D-BBR, it should be noticed that there is only one possible transverse polarization state of an electromagnetic wave in 2D since it is not possible to rotate an electrical field vector in 2D. Modifying equation (7) by using this fact and the identity of $d^2 \mathbf{p} \rightarrow 2\pi p dp$, the energy of 2D-BBR can be obtained as,

$$U = A \frac{1}{(2\pi\hbar)^2} \int_0^{\infty} \frac{cpd^2 \mathbf{p}}{\exp(cp/k_b T) - 1} \quad (9a)$$

$$U = A \frac{2\pi k_b^3 T^3}{(2\pi\hbar c)^2} \int_0^{\infty} \frac{x^2 dx}{\exp(x) - 1} = \frac{\xi(3)k_b^3}{\pi \hbar^2 c^2} AT^3. \quad (9b)$$

Equation (9b) is the same with the second term in equation (5). Therefore, it is understood that temperature dependent thermodynamic properties caused by the Casimir effect at finite temperature originates from the thermodynamic properties of 2D-BBR. Equation (6) also represents the entropy of 2D-BBR. It can be derived by using the similar way in the derivation of energy of 2D-BBR.

For $T \gg T_C$, pressure, energy and entropy quantities are given as the following forms under the zero external temperature assumption

$$p \cong \frac{\pi^2 k_b^4}{45\hbar^3 c^3} T^4, \quad (10)$$

$$U \cong \frac{\pi^2 k_b^4}{15\hbar^3 c^3} T^4 Ad, \quad (11)$$

$$S \cong \frac{4\pi^2 k_b^4}{45\hbar^3 c^3} T^3 Ad. \quad (12)$$

Equations (10)-(12) are the classical expressions of 3D-BBR [12]. Therefore, it is seen that thermodynamic properties of the system consist of the thermodynamic properties of 3D-BBR when $T \gg T_C$.

By using equations (4)-(6) and (10)-(12), heat and work exchange expressions can be derived for isothermal, isobaric, isochoric and isentropic processes at low and high temperature limits as follows:

i) Isothermal Process (T is constant)

- For $T < T_C$:

$$Q_{if} = \frac{3}{2} \frac{\xi(3)k_b^3}{\pi \hbar^2 c^2} T^3 (A_f - A_i) \quad (13)$$

$$W_{if} = (A_f - A_i) \frac{1}{2} \frac{\xi(3)k_b^3}{\pi \hbar^2 c^2} T^3 + \frac{\pi^2 \hbar c}{720} \left(\frac{A_f}{d_f^3} - \frac{A_i}{d_i^3} \right) \quad (14)$$

- For $T \gg T_C$:

$$Q_{if} = \frac{4\pi^2 k_b^4}{45\hbar^3 c^3} T^4 (A_f d_f - A_i d_i) \quad (15)$$

$$W_{if} = \frac{\pi^2 k_b^4}{45\hbar^3 c^3} T^4 (A_f d_f - A_i d_i) \quad (16)$$

ii) Isobaric Process (P is constant)

- For $T < T_C$:

In this case, d is also a constant due to equation (4). Heat exchange expression is

$$Q_{if} = \frac{3}{2} \frac{\xi(3)k_b^3}{\pi \hbar^2 c^2} \int_i^f (2AT^2 dT + T^3 dA). \quad (17)$$

In case that both A and T are variable, the relation between A and T should be known since Q_{if} depends on this relation. Therefore, Q_{if} is calculated by equation (17) when $A(T)$ is known. On the other hand, it is possible to give a solution for two simple case:

If A is a constant, then

$$Q_{if} = \frac{\xi(3)k_b^3}{\pi\hbar^2 c^2} A (T_f^3 - T_i^3) \quad (18)$$

$$W_{if}=0 \quad (19)$$

If T is a constant, then

$$Q_{if} = \frac{3}{2} \frac{\xi(3)k_b^3}{\pi\hbar^2 c^2} T^3 (A_f - A_i) \quad (20)$$

$$W_{if} = \frac{1}{2} \frac{\xi(3)k_b^3}{\pi\hbar^2 c^2} T^3 (A_f - A_i) \quad (21)$$

- For $T \gg T_C$:

In this case, T is also a constant due to equation (10). Therefore equations (15) and (16) are valid for heat and work exchange expressions respectively.

iii) Isochoric process ($V=Ad$ is constant)

$$W_{if}=0 \quad (22)$$

- For $T < T_C$:

$$Q_{if} = \frac{\xi(3)k_b^3}{\pi\hbar^2 c^2} A (T_f^3 - T_i^3) \quad (23)$$

- For $T \gg T_C$:

$$Q_{if} = \frac{\pi^2 k_b^4}{15\hbar^3 c^3} Ad (T_f^4 - T_i^4) \quad (24)$$

iv) Isentropic process (S is a constant)

$$Q_{if}=0 \quad (25)$$

- For $T < T_C$:

In this case, the quantity of AT^2 is a constant due to equation (6).

$$W_{if} = \frac{\xi(3)k_b^3}{\pi\hbar^2 c^2} T_i^3 A_i \left[1 - \sqrt{\frac{A_i}{A_f}} \right] + \frac{\pi\hbar^2 c^2}{720} \left(\frac{A_f}{d_f^3} - \frac{A_i}{d_i^3} \right) \quad (26)$$

- For $T \gg T_C$:

In this case, the quantity of $T^3 Ad$ is a constant due to equation (12).

$$W_{if} = \frac{\pi^2 k_b^4}{15\hbar^3 c^3} A_f d_f T_f^3 (T_f - T_i) \quad (27)$$

4. THERMODYNAMIC ANALYSIS OF A CARNOT POWER CYCLE BASED ON THE CASIMIR EFFECT AT FINITE-TEMPERATURE

T - S diagram of a Carnot power cycle is shown in Figure 2. Since the Carnot efficiency is independent from the physical properties of working fluid, efficiency remains unchanged. Therefore, net work output is $W = Q_H \eta_c = Q_{23} \eta_c$.

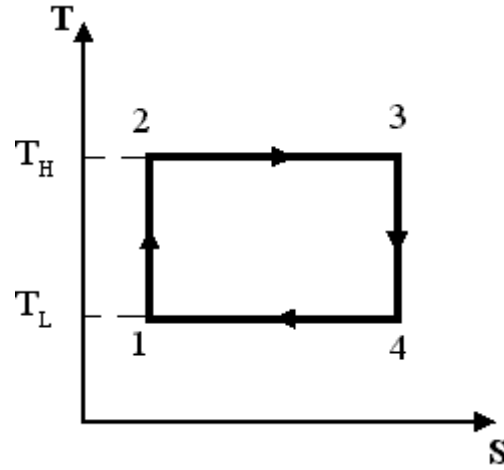


Figure 2: T - S diagram of a Carnot power cycle.

By using equations (13) and (15) for Q_{23} , maximum work and maximum work density are obtained for low and high temperature limits as follows:

- Case I: Low temperature limit ($T_H < T_C$)

The condition of $T_H < T_C$ restricts the maximum distance between the plates, d_H , as $d_H < \pi\hbar c / k_b T_H$. If λ_L is defined as a parameter for low temperature limit, one can write $d_H = \lambda_L \pi\hbar c / k_b T_H$ and $\lambda_L < 1$. Using equation (13), one can write

$$W = \frac{3}{2} \frac{\xi(3)k_b^3}{\pi\hbar^2 c^2} T_H^3 (A_3 - A_2) \eta_c \quad (28)$$

By considering equations (6) and (13), it is seen that $A_2 < A_3 < A_4$, $A_2 < A_1$ and $A_3 T_H^2 = A_4 T_L^2$. Since A_2 is the lowest area while A_4 is the highest one, they can symbolized as $A_2 = A_L$ and $A_4 = A_H$. Thus, equation (28) becomes

$$W = \frac{3}{2} \frac{\xi(3)k_b^3}{\pi\hbar^2 c^2} T_H^3 A_H \left(\tau^3 - \frac{1}{r_A} \right) (1 - \tau) \quad (29)$$

where r_A is area ratio defined as $r_A = A_H / A_L$ and τ is temperature ratio defined as $\tau = T_L / T_H$. According to equation (29), W reaches to its maximum value when r_A goes to infinity and τ goes to 3/4. This maximum is the ultimate value of W . Therefore, maximum work is

$$W_{Max} = \frac{81}{512} \frac{\xi(3)k_b^3}{\pi\hbar^2 c^2} T_H^3 A_H \quad (30)$$

If maximum work density, $(W_d)_{Max}$, is defined as maximum work per maximum volume, $(W_d)_{Max}$ is

$$(W_d)_{Max} = \frac{W_{Max}}{A_H d_H} = \frac{81}{512} \frac{\xi(3)k_b^4}{\pi^2 \hbar^3 c^3} \frac{T_H^4}{\lambda} \quad (31)$$

- Case II: High temperature limit ($T_L \gg T_C$)

The condition of $T_L \gg T_C$ leads to the condition of $d_L \gg \pi\hbar c / k_b T_L$. If λ_H is defined as a parameter for high temperature limit, one can write

$d_L = \lambda_H \pi \hbar c / k_b T_L$ and $\lambda_H \gg 1$. Using equation (15), work is expressed as

$$W = \frac{4\pi^2 k_b^4}{45\hbar^3 c^3} T_H^4 (V_3 - V_2) \eta_c \quad (32)$$

where V is the volume defined as $V=Ad$. According to equations (12) and (15), one can write that $V_2 < V_3 < V_4$, $V_2 < V_1$ and $V_3 T_H^2 = V_4 T_L^2$. Since V_2 is the lowest volume while V_4 is the highest one, they can symbolized as $V_2 = V_L$ and $V_4 = V_H$. Therefore, equation (32) can be re-written as

$$W = \frac{4\pi^2 k_b^4}{45\hbar^3 c^3} T_H^4 V_H \left(\tau^3 - \frac{1}{r_V} \right) (1 - \tau) \quad (33)$$

where r_V is volume ratio defined as $r_V = V_H / V_L$. The condition for maximum possible work is the same with that of equation (29), $r_V \rightarrow \infty$ and $\tau \rightarrow 3/4$. Consequently, maximum work for high temperature limit is

$$W_{Max} = \frac{3\pi^2 k_b^4}{320\hbar^3 c^3} T_H^4 V_H \quad (34)$$

$(W_d)_{Max}$ is easily obtained from equation (34) as

$$(W_d)_{Max} = \frac{W_{Max}}{V_H} = \frac{3\pi^2 k_b^4}{320\hbar^3 c^3} T_H^4 \quad (35)$$

To give an idea about the magnitude of maximum work and maximum work density, numerical results are obtained for low and high temperature limits by choosing the values of T_H and A_H as $T_H=1000$ K and $A_H=1$ m². By using equation (31), maximum work density and maximum distance at low temperature limit are obtained in term of λ_L as

$$(W_d)_{Max} \cong \frac{22}{\lambda_L} [\mu J / m^3] \quad (36)$$

$$d_H \cong 7200 \lambda_L [\text{nm}] \quad (37)$$

Therefore, maximum work is calculated by using equations (36) and (37) as

$$W_{Max} = (W_d)_{Max} A_H d_H \cong 0.16 \text{ nJ} \quad (38)$$

If λ_L is taken as $\lambda_L=0.01$, then equations (36) and (37) give the following results

$$(W_d)_{Max} \cong 2200 \mu J / m^3 \quad (39)$$

$$d_H \cong 72 \text{ nm} \quad (40)$$

Similarly, by using equation (35), maximum work density at high temperature limit are calculated as

$$(W_d)_{Max} \cong 106 [\mu J / m^3]. \quad (41)$$

By considering $d_L = \lambda_H \pi \hbar c / k_b T_L$ and noting that $\tau = 3/4$, the minimum distance, d_L , is expressed in term of λ_H as

$$d_L \cong 9603 \lambda_H [\text{nm}] \quad (42)$$

If λ_H is taken as $\lambda_H=100$, then equation (42) give the following result

$$d_L \cong 960 \mu m \quad (43)$$

In this case, there is no any limitation on d_H . If $d_H=1$ m, then maximum work is obtained by equation (41) as

$$W_{Max} = (W_d)_{Max} A_H d_H \cong 106 \mu J \quad (44)$$

5. CONCLUSION

Heat and work expressions derived in Section 3 can be used to establish the thermodynamic models for other cycles (like as Brayton, Otto, etc.) based on the Casimir effect.

By considering the results obtained in this work, it can be said that the heat engines based on the Casimir effect can barely produce the net work output per cycle in nJ or μJ scale even for the most ideal case. Temperature dependent thermodynamic properties of the Casimir effect results from the thermodynamic properties of BBR. Therefore, working gas consists of BBR. Low temperature properties of the Casimir effect deal with 2D-BBR while its high temperature properties deal with 3D-BBR. The energy density of BBR is too small at common temperatures, in comparison with the ordinary gases. Thus, net work output of a heat engine based on the Casimir effect is too small than that of the engine working with ordinary gases. For this reason, they do not seem to be used for macro scale power production.

From equations (31) and (35), it is seen that work density depends on the parameter λ_L besides T_H in low temperature limit while it depends on only T_H in high temperature limit. Therefore, it is possible to increase the work density by decreasing the value of λ_L when the engine is designed to operate in low temperature limit. This property provides an advantage when the work density is important. On the other hand, if the net work output is important, the engine should be designed to operate in high temperature limit. Because, as can be seen by comparing the equations (38) and (44), net work output in low temperature limit is much less than the net work output in high temperature limit. In high temperature limit, however, distance between the plates is too large in comparison with the distance in low temperature limit.

Consequently, it seems that the heat engines based on the Casimir effect promise to produce only micro-scale power production even if some of their sizes (such as surface area) in macro-scale. However, analyses of such heat engines provide an insight about the physical details of working mechanism of this kind of engines.

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Nomenclature

A	: Surface area of the mobile plate.
A_L	: Low value of A
A_H	: High value of A

c	: Speed of light in the vacuum.
d	: Distance between the plates
d_H	: High value of d
d_L	: Low value of d
d	: Differential operator
$\hbar$	: Reduced Planck's constant
k_b	: Boltzmann's constant
p	: Pressure, scalar value of momentum vector
$\mathbf{p}$	: Momentum vector
Q	: Heat transferred
r_A	: Area ratio defined as $r_A = A_H / A_L$
r_V	: Volume ratio defined as $r_V = V_H / V_L$
S	: Entropy
T	: Temperature
T_c	: Critical temperature defined as $T_c = \pi \hbar c / k_b d$
T_H	: High temperature
T_L	: Low temperature
U	: Total energy
V	: Volume
W	: Work
W_d	: Work density
$(W_d)_{Max}$	: Maximum work density
W_{Max}	: Maximum work
η_C	: Carnot efficiency
λ_L	: Parameter for low temperature limit
λ_H	: Parameter for high temperature limit
τ	: Temperature ratio defined as $\tau = T_L / T_H$

Subscripts

i	: Initial state
f	: Final state