DIAPHRAGM STIRLING ENGINE DESIGN

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ABSTRACT

This paper describes the principal design features and the dynamic and thermodynamic analysis of a prototype Stirling engine intended for use as a low cost and simple prime mover in natural gas-fired small-scale power generator system. The engine is of the diaphragm type with a displacer separated from the cylinder by a narrow gas-bearing gap. The combination of the rubber diaphragm and the absence of sliding seals lead to very quiet operation and lower frictional losses than for engines with piston rings. The engine operates using air as working fluid which places less stringent mechanical constraints on containing the working fluid than using a lighter gas such as helium. A detailed thermal analysis was carried out using a second order computer model which computes temperature and pressure variation throughout the engine working space, heat supplied to and rejected from the engine, work output as calculated from the PV diagram, and efficiency of the cycle.
Finally a proof-of-concept prototype has been built and tested to demonstrate the feasibility of the system.

KEYWORDS

Stirling engine, free piston, domestic combined heat and power.

1 INTRODUCTION

With today's increasing concerns about global warming and the impact of pollutants arising from fossil fuels and the consequent need to improve energy utilisation there has been a pressing demand for an electric power generator and cogenerator package for domestic application. Recent studies show that the demand exists for a device of 0.5 to 5kW [1,2]. However to date small scale electricity generating systems are built around internal combustion engines, wind turbines, or Photo-voltaic systems that are either noisy, weather or/and location dependent. Currently there is concerted research effort to develop a simple, reliable, and low cost power generator using Stirling heat engine.

Stirling heat engine has long been proposed as a suitable and efficient prime mover for converting heat energy to mechanical form in power generator units; but more recently the main stimulus has been the development of Domestic Combined Heat and power (DCHP) systems.

In the idealised form of Stirling engine heat is both supplied to and rejected from the engine during isothermal expansion and compression processes, respectively. The cycle is completed by two constant volume processes, during which the heat transported by the working fluid is stored and released by a regenerative heat exchanger. Theoretically, the cycle efficiency may reach the maximum achieved by the Carnot cycle (η_c=1-T_c/T_e).

In the last few decades significant advances in Stirling engine design has been made with the invention by William Beale of the Free-Piston Stirling Engine (FPSE). Unlike kinematic Stirling engines, in which the pistons (i.e., a power piston and a displacer) are connected to the load by a crank mechanism, FPSE reciprocating elements have no direct linkage, their motion relying only on gas pressure, reciprocating masses and spring stiffness [3]. These devices can be hermetically sealed and pressurised, permitting the use of working fluids such as hydrogen and helium to achieve higher specific power output.

The Stirling engine described in this paper introduces innovative changes to the design concept originally proposed by Cooke-Yarborough [4,5] at Harwell, known as the Thermo-Mechanical Generator (TMG). TMG uses a flexible metal diaphragm as a power piston and hence eliminating sliding seals and rubbing surfaces, however, metal fatigue resulting from stress and strain of the diaphragm material restricted its amplitude of deflection to less than 0.5% of its diameter and hence making it uneconomical for further development.

In the present engine design the diaphragm piston is made from rubber material of suitable hardness grade, that combines high resilience, large deflection, and low material hysteresis loss. Hence by eliminating deflection restriction, prospect of higher swept volume and power output can be achieved. In addition, accurate thermal modelling of the engine lead to design of effective external heat exchangers (i.e., heater, cooler, and regenerator) with low pressure drop

2 ENGINE DESIGN AND LAYOUT

The diaphragm Stirling engine is made of two working space volumes (i.e., expansion and compression space) which are maintained at two temperature levels and varied cyclically out of phase (see Figure 1.). These spaces are coupled through three heat exchangers namely heater, regenerator and cooler. The heater is made of plain stainless steel tubes and is located on the expansion space side of the regenerator. Sensible heat energy released in the combustion chamber is transferred though the heater tubes wall to the working fluid in the engine at temperature of 450 to 500°C. The cooler is made of copper tubes and located on the compression space side of the regenerator and through which heat is exchanged between the working fluid and the heat sink at temperature of 30 to 50°C. The regenerator is made of fine stainless steel meshes (150 mesh/in) and packed in an annulus gap built around the engine cylinder. Its main function is to temporarily store heat energy carried by the working fluid as it flows from the expansion space to the compression space and release it back to the working fluid as it reverses direction. The regenerator accounts for most of the flow frictional loss and also affects significantly temperature variation in the expansion and compression space and hence thermal efficiency of the engine.
The engine also comprises two moving components, the displacer piston and the flat rubber diaphragm.

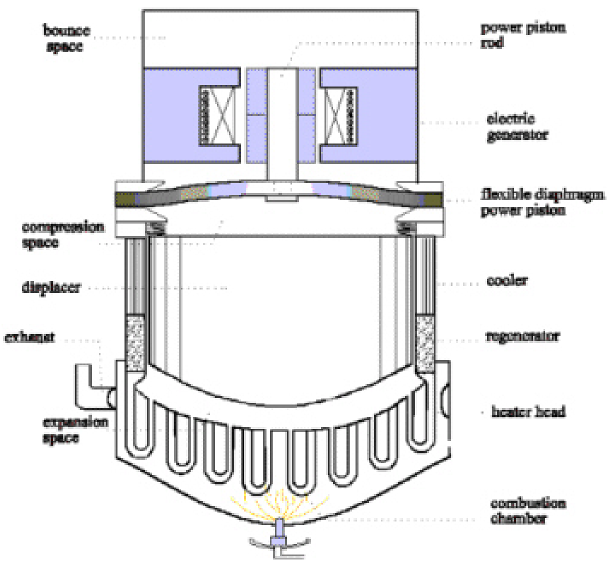

Figure 1 Schematic diagram of the engine

2.1 DISPLACER

The displacer piston is housed in the engine cylinder between the expansion space and the compression space and hence is exposed to large temperature gradient. To minimise heat transfer by conduction from the expansion space to the compression, the displacer was made from a sealed hollow cylinder using a combination of aluminium alloy, at the cold end, and low thermal conductivity material at the hot end. The low thermal conductivity material (ceramics) forms a heat conduction barrier of low thermal conductivity (λ=1 W/m K) and stable thermal characteristics at high temperature (900°C). In addition, the displacer structure needs to be of lightweight and rigid to withstand working fluid pressure force.

2.2 DIAPHRAGM

The diaphragm is made of a flat rubber disk clamped at its circumference to the engine cylinder and has no direct mechanical linkage to the displacer. The prime function of the diaphragm is to convert working fluid pressure variation into mechanical oscillation. The diaphragm oscillation is then transmitted through a reciprocating shaft to a linear electric generator, which in turn converts this mechanical oscillation into electrical form.
The design of the diaphragm involves selection of a suitable rubber material, calculation of critical stress and strain forces, natural frequency of oscillation, and energy loss within the material. In this research work two rubber compound material were examined - Silicon rubber and natural rubber. Although silicon rubber has superior characteristics (i.e., higher operating temperature and lower damping losses), natural rubber was chosen for its low cost. A vulcanised natural rubber material of 50-60 IRHD[1] was found to be most suitable as it combines high resilience and lower damping losses. It also has stable operating characteristics (i.e., Young's modulus and stiffness) over temperature range of 0°C to 50°C and frequency below 1000 Hz. In this design Young's modulus of the selected diaphragm material was approximately equals 2.5 10^6 N/m^2.
The action of the working fluid pressure upon the diaphragm surface area causes the diaphragm to deflect and hence change of the working space volume. The relationship between the pressure difference across the diaphragm and the volume displaced can approximately be given by [6]:

$$p = \frac{8\, E_p \qquad t^4}{\pi\, R^6 (1 - \mu_p^2)} \left(\frac{V_{sw}}{t}\right) \tag{1}$$

One of the design requirements was to achieve a long life operation and hence the diaphragm should perform more over 10^{10} cycles before any deterioration of its structure occurs. Available literature on fatigue life of rubber material show that upon reducing the dynamic stress below a certain value, known as the critical stress level and which is $0.3x10^6$.N/mm, the probability of fatigue failure of the material to occur is very small.
For a flat diaphragm clamped at its edge and with a mass attached at its centre, the natural frequency of oscillation can be given as [6]: -

$$\omega = \sqrt{\frac{K_p}{m_{peff} + m_r}} \tag{2}$$

In practice however it was found that the operating frequency of the diaphragm is lower than that predicted by equation 2 because of the effect of the surrounding working fluid, imperfect edge clamping, and wrinkling of the diaphragm.
Finally, the dynamic oscillation of the diaphragm causes the diaphragm to heat internally because of hysteresis

[1] International Rubber Hardness Degree.

losses that appears as heat in the bulk of the diaphragm. By assuming that the oscillation amplitude of the diaphragm is small compared to its thickness, the amount of heat energy generated within the material per cycle can be given as:

$$W_{diss} = \pi \zeta \, \omega \, X_p^2 \tag{3}$$

Where ζ can be determined using mean logarithmic decrement method.
From equation 3 it can be seen that the heat generated within the diaphragm material is greatly influenced by the diaphragm oscillation amplitude. The diaphragm dimensions were calculated by solving equation 1,2, and 3 simultaneously so that optimum parameters were obtained.

2.3 POWER OUTPUT

A preliminary estimate of the engine shaft power can be calculated from West's equation, given as [7]:

$$W = 0.025 \, p_m f_r V_{sw} \frac{T_e - T_c}{T_e + T_c} \tag{4}$$

For a known set of operating parameters such as temperatures, mean pressure, and frequency, equation 4 can be used to yield broad estimate of the engine size. This equation does not however quantify heat losses involved in the engine (i.e., heat loss due to conduction, convection, shuttle, and friction). Therefore a computer model was developed to optimise heat transfer components (heater, regenerator, and cooler) of the engine and hence minimise energy losses due to working fluid pressure drop, heat conduction, and heat convection.

3 OUTLINE OF THERMAL MODELLING

Thermal modelling of the engine was conducted using third order mathematical analysis based on finite cell method [8,9]. The method consists in subdividing the working fluid flow-passages, that is heat exchangers and connecting ports, into six homogenous control volumes; namely expansion space, heater, cooler and compression space and two control volumes for the regenerator. In this analysis it was assumed that perfect gas law applies in each control volume and the flow is one-dimensional throughout the engine. Figure 2.shows heat balance of a typical control volume cell.
Each control volume was defined by a set of differential equation of conservation of energy, mass, and momentum, as follows: -

- Energy conservation

$$\frac{dQ_j}{dt} - Q_{dissj} + C_p\left(T_i m_i - T_{i-1} m_{i-1}\right) = C_v \frac{d}{dt}\left(m_j T_j\right) + dW_j \tag{5}$$

- Mass conservation

$$\sum_1^6 m_j = m_o \tag{6}$$

- Mass continuity

$$\frac{dm_j}{dt} = m_i - m_{i-1} \tag{7}$$

- Momentum conservation

$$p_i - p_{i-1} = f_j \frac{L_j}{D_{hj}} \rho_j u_j^2 \frac{u}{|u|} \tag{8}$$

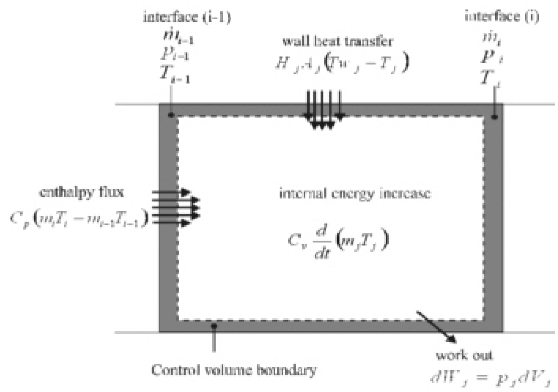

Figure 2 Typical control volume heat balance

Equation (5), (6), (7) and (8) were set up for every control volume and then solved simultaneously using a second order Runge-Kutta method. Prior to running the program the motion of the displacer and diaphragm, as function of time, operating frequency, working fluid mass, fluid type, and the geometry of the system have to be specified. In the analysis it was assumed that expansion and compression control volumes vary adiabatically and their initial temperatures are not known in advance hence the simulation model is a boundary-value problem. However due to cyclic behaviour of the working fluid in the engine, it was suggested by Urieli [8] that it is still possible to formulate the model as an initial-value problem by assigning an appropriate initial conditions and integrating the differential equations until a steady state was reached. A convergence acceleration technique was applied at the end of each cycle to obtain the steady state temperature profile in a shorter computer processing time. Time dependent parameters of the engine computed over one complete cycle using this model are working fluid temperature variation in the engine, pressure fluctuation, volume swept in the expansion and compression space, and PV diagram of the engine. These results are shown in Figure 3, 4, 5 and Table 1.
Figure 3 shows temperature variation in each compartment of the engine through one complete cycle. It can be seen that the temperature of the working fluid in both expansion and compression space undergoes a considerable variation despite a careful process of regenerator design. Figure 4 shows pressure variation in the expansion and compression space of the engine and the two pressure waveforms are nearly in phase, which underpins low pressure losses in the heat exchangers. Figure 5 shows volume swept by the displacer and diaphragm and a 90° phase-shift was

assumed. Figure 6 show a plot of the PV diagram which enclosed area is a measure of the power output of the engine

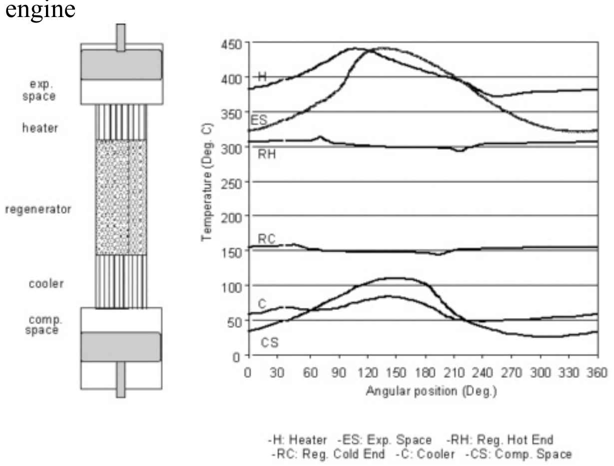

Figure 3 Working fluid temperature variation

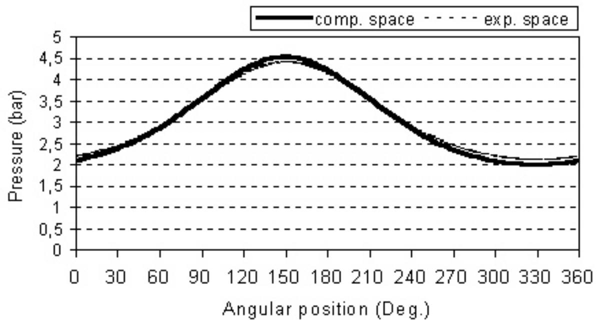

Figure 4. Pressure Variation

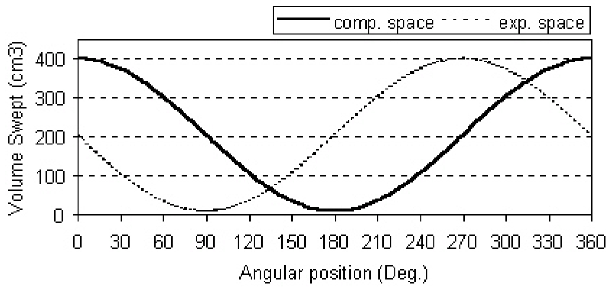

Figure 5. Volume variation o f compression and expension space

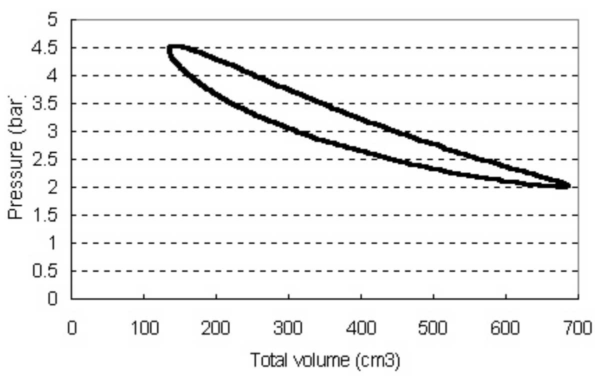

Figure 6. Pressure-Volume diagramme of the engine

4 DYNAMIC ANALYSIS

Dynamically the engine can be represented by three oscillating masses (i.e., displacer, diaphragm and casing mass) Each of the component is subject to a driving pressure force, an inertia force and a damping force. The diaphragm driving force results from the pressure difference between the engine working space and bounce space (i.e., the space that houses the electric generator). The displacer however is driven by pressure difference between the compression and expansion space. The frequency of operation of the entire system operates near displacer natural frequency, which is equals to $(k/M)^{½}$, and the diaphragm frequency which is given by equation 1. In addition the gap between the cylinder and the displacer was made very small (<0.3mm) and the working fluid present in the gap acts as an effective gas bearing and hence prevents contact between the moving surfaces and further reduces friction losses.

5 EXPERIMENTAL INVESTIGATION

Figure 7 shows a picture of the built engine test rig. The system comprises the engine and a linear electric generator and associated instruments. The engine heater is located inside and forms part of the combustion chamber and tap water was circulated in the cooler jacket of the engine to provide cooling. The tests were performed at near atmospheric pressure and air used as the working fluid. Instruments were used to measure working pressure variations, compression and expansion space temperatures, diaphragm and displacer amplitudes and frequency. The engine functioned for over 100 hours without any sign of deterioration to the engine components including the diaphragm.

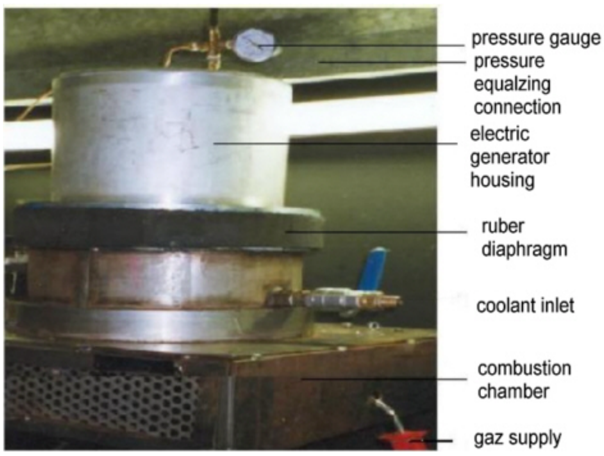

Figure 7 Rubber Diaphragm Stirling engine test rig

The prototype proved to be capable of self-starting without the aid of a starter motor and hence eliminating complicated control system. It was also shown that the engine is very quiet in operation (barely audible at a distance of 1m) and capable of operating at low temperature (350°C). Throughout the tests the engine operated at constant frequency of 33 Hz and displacer and diaphragm amplitudes of 4 mm and 8 mm respectively. However inefficient linear electric generator used to convert shaft power to electricity meant that it was very difficult to measure accurately the power output. Table 1 shows the engine operating parameters.

Table 1: Design parameters of the diaphragm Stirling engine

Diaphragm Young's modulus	2.5 MN/m^2
Diaphragm material Poisson's ratio	0.5
Diaphragm density	1000 kg/m^2
Diaphragm maximum operating temperature	140°C
Diaphragm maximum dynamic stress	0.3 MN/m^2
Cylinder diameter	250 mm
Diaphragm diameter	270 mm
Diaphragm thickness	35 mm
Maximum diaphragm amplitude	10 mm
Displacer length	110 mm
Displacer mass	3.2 kg
Annular gap between displacer and cylinder	0.3 mm
Stiffness of each spring supporting displacer	34.5 N/mm
Maximum displacer amplitude	5 mm
Regenerator outside diameter	330 mm
Regenerator height	30 mm
Regenerator mesh diameter	0.09 mm
Porosity of mesh	67%
Cooler jacket height	170 mm
Cooler tubes diameter	3.2 mm
Heater tubes diameter	4.1 mm

6 CONCLUSIONS

A proof-of-concept prototype Stirling engine intended for domestic combined heat and power has been design, built and tested. It has been demonstrated that by using rubber diaphragm as power piston the engine mechanical assembly was made simple and easy to manufacture. The prototype satisfied the condition of self-starting, quiet in operation, low vibration, and fuel from bottled gas or mains can be used. Therefore the design can be extended to achieve high power output by pressurising the engine to higher pressure.

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NOMENCLATURE

C_p	working fluid specific heat capacity at constant pressure	(J/kg K)
C_v	working fluid specific heat capacity at constant volume	(J/kg K)
D_h	hydraulic diameter	(m)
E	Young’s modulus	(N/m^2)
f_r	engine operating frequency	(Hz)
f	friction factor	(-)
K	spring stiffness	(N/m)
m	mass	(kg)
P	pressure	(N/m^2)
Q	heat energy	(J)
R	Diaphragm radius	(m)
t	diaphragm thickness	(m)
T	temperature	(K)
U	working fluid velocity	(m/s)
Vsw	swept volume	(m^3)
W	power output	(W)
X	diaphragm oscillation amplitude	(m)
ζ	damping coefficient	(Ns/m)
μ	Poisson's ratio	(-)
ρ	density	(kg)
ω	angular frequency	(rd/s)

Subscripts

c, e	compression and expansion space respectively.
diss	dissipation.
j,i	control volume and control volume boundary interface respectively.
m, o	mean and total value respectively.
eff, p, r	effective, diaphragm and reciprocating mass respectively.